M 408M Exam 2 Practice Problems

1. Find a vector that is orthogonal to both $v = \langle 1, 4, 3 \rangle$ and $u = \langle -2, 2, 1 \rangle$. Use this to find an equation of a plane that contains u and v, assuming both u and v start at the origin.

2. Find a vector equation for the line that goes through $P(-1, 1, 0)$ and $Q(1, 4, 6)$. At what points does this line intersect the sphere centered at P with radius 7?

3. Find parametric equations for the line that goes through $P(1, 2, 3)$, is parallel to $3x + 2y + z = 5$, and perpendicular to the line $\langle 2 + t, 3 - t, 1 - t \rangle$.

4. Let $r(t) = \langle 5\sin(\pi t), 2e^t, 3\arctan(\sqrt{3}t) \rangle$. First, identify what type of object (a), (b), (c), and (d) are (i.e., are they scalars, vectors, scalar functions, vector functions, etc.), then find them.
   (a) $\lim_{t \to 0} r(t)$
   (b) $\dfrac{d}{dt} r(t)$
   (c) $\displaystyle\int_0^t r(s)\, ds$
   (d) $\|r'(t)\|$

5. Find the vector function that describes the intersection between the plane $x = 6$ and the sphere $x^2 + y^2 + z^2 = 100$. Identify the curve, and find its circumference.

6. Determine whether the limit
$$\lim_{(x,y) \to (0,0)} \frac{x^2 + x^2y^2 + y^2}{x^2 + y^2}$$
exists. If it does exist, find the value of the limit.

7. Determine whether the limit
$$\lim_{(x,y,z) \to (0,0,0)} \frac{x^2 + y^2 + z^2}{\sin(x)e^{x+y}z^6}$$
exists. If it does exist, find the value of the limit.