M 408L Practice Exam 3

This exam is meant to (roughly) be the same length as the actual exam (i.e., $\sim$ 20 questions). I would recommend starting a stopwatch right before you attempt the exam – try to complete this exam in 1 sitting as well. Whenever you complete a problem, you can hit the "lap" button on your stopwatch to keep track of how long each question takes.

1. Determine if the following series absolutely converges, conditionally converges, or diverges:

$$\sum_{k=1}^{\infty} \cos(k\pi) \cdot \frac{k^3}{(k+5)^4 \ln(k)}$$

2. Determine if the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{3n^4}{6n^5 \ln(n) + 7}$$

3. Determine if the following series converges or diverges:

$$\sum_{m=1}^{\infty} \frac{(\ln(m))^2}{m^3 + 5}$$

4. Let g be a continuous, positive, and decreasing function on $[1, \infty)$. Order A, B, and C from least to greatest, where

$$A = \int_1^{10} g(x)\, dx, \quad B = \sum_{n=1}^{10} g(n), \quad C = \sum_{n=2}^{10} g(n).$$

(hint: draw a picture for this problem)

5. Determine if the following series converges or diverges:

$$\sum_{n=2}^{\infty} \frac{n^{1/3} + 5}{\sqrt{n^2 + 50n}}$$

6. Determine if the following series absolutely converges, conditionally converges, or diverges:

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{n^n + 5}$$

7. Determine if the following series absolutely converges, conditionally converges, or diverges:

$$\sum_{n=0}^{\infty} (-1)^n \tan^{-1}\left(\frac{n}{n+2}\right)$$

8. Determine if the following series absolutely converges, conditionally converges, or diverges:

$$\frac{2}{4} - \left(\frac{4}{7}\right)^2 + \left(\frac{6}{10}\right)^3 - \left(\frac{8}{13}\right)^4 + \left(\frac{10}{16}\right)^5 - \left(\frac{12}{19}\right)^6 + \dots$$

   (note: the main challenge in this problem is converting this series into our usual notation. Try to find a pattern in the numerators and the denominators of each term)

9. Evaluate the following integral as a power series:

$$\int_0^2 \sin(x^2)\,dx$$

10. Write the following function as a power series:

$$2\cos(x) + 4\sin(3x)$$

11. Find the interval and radius of convergence of the following power series:

$$\sum_{n=1}^{\infty} n^{15}(x+5)^n$$

12. Find the interval and radius of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{3n(2x-1)^n}{4^n}$$

13. Evaluate the following derivative as a power series:

$$\frac{d}{dx}\ln(1+4x^2)$$

14. Write the following function as a power series:

$$\frac{1}{1-3x^3}.$$

   What is the interval of convergence of your power series?

15. Write the first 2 nonzero terms of the Taylor series of the function $f(x) = \cos(x^2)$ centered at $a = \sqrt{\pi}$.

16. Determine if the following series absolutely converges, conditionally converges, or diverges:

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^{3/2} + k^{1/2}}{k^2 + 3k + 1}$$

17. Use the degree 2 Taylor polynomial of $f(x) = e^{x^2}$ centered at $a = 1$ to approximate e^4 (your answer will look like a number times e).

18. Write the Maclaurin series of $x^2 \tan^{-1}(\frac{1}{2}x^3)$.

19. Find the radius and interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{n!(x-3)^n}{50^n}$$