M 408L Practice Exam 2

This exam is meant to (roughly) be the same length as the actual exam (i.e., ~ 20 questions). I would recommend starting a stopwatch right before you attempt the exam – try to complete this exam in 1 sitting as well. Whenever you complete a problem, you can hit the "lap" button on your stopwatch to keep track of how long each question takes.

1. Determine if $\displaystyle\int_0^1 \frac{1}{x^5}\,dx$ converges or diverges (if it converges, then **do** find the value of the integral).

2. Determine if the following sequence a_n converges or diverges, and if it converges, identify the limit:
$$a_n = \frac{(-1)^{n+1}\cos(n) + n}{2\pi n^2}$$

3. Evaluate
$$\int \frac{x^2}{(x-5)(x+1)^2}\,dx$$
(note: this is the hardest exercise on this exam. Do not be dismayed if this one takes you a while – on the real exam, the partial fraction problems are unlikely to be any more difficult than this).

4. Compute
$$\int\int_D (ye^{x+y})\,dxdy,$$
where $D = \{(x,y) : 0 \le y \le 2, 0 \le x \le 3\}$.

5. Evaluate
$$\int_{\ln 2}^{\ln 5} \frac{e^x}{e^{2x}-1}\,dx$$

6. Determine if $\displaystyle\int_3^\infty \frac{|\cos(x)\sin(x)|}{x^5}\,dx$ converges or diverges (if it converges, do not find the value of the integral).

7. Evaluate
$$\int_{-1}^0 \frac{x^3+2x^2+5}{x^2+x-2}\,dx$$

8. Find the volume between the function $f(x,y) = 3(x^2+y^2)$ and the region in the xy plane bounded by the curve $x = y^2$ and the line $x = 4$.

9. Let $f(x,y) = (x+y)^2 \ln(y)$. Find $f_{yx}(x,y)$ and $f_{xy}(x,y)$.

10. Determine if $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{|x|}}\,dx$ converges or diverges (if it converges, then **do** find the value of the integral).

11. Compute

$$\int\int_D xy\,dydx,$$

where $D = \{(x,y) : 0 \le x \le \sqrt{y}, 1 \le y \le 2\}$.

12. Determine if the following sequence a_n converges, and if it converges, identify the limit:

$$a_n = \sin\left(\frac{3\pi n^2}{1+n-2n^2}\right)$$

13. Determine if the following sequence a_n converges, and if it converges, identify the limit:

$$a_n = \frac{n^2}{\ln(n+1)}$$

14. Determine if the following series converges, and if it converges, identify the limit:

$$\sum_{n=2}^{\infty} 3^{n-1} \cdot 5^{2-2n}$$

15. Compute

$$\int\int_D (x^2+xy)\,dydx,$$

where D is the triangle in the xy plane with vertices at $(1,0)$, $(0,1)$, and $(1,1)$.

16. Given the fact that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, explain why the following series diverges:

$$\sum_{n=1}^{\infty} \frac{3n}{n^2-5}$$

17. Determine if $\displaystyle\int_1^\infty \frac{\ln x}{x+1}\,dx$ converges or diverges (if it converges, do not find the value of the integral).

18. Change the order of integration for the following integral:

$$\int_0^3 \int_2^{5-x} 6\sin(e^{xy})\,dydx.$$

19. Below is the contour plot of a function $f(x,y)$. Determine the signs (meaning, are they positive, negative, or zero) of f_x and f_y at point P (in blue). Do the same at point Q (in red) as well.