M 408L Practice Exam 1

The actual exam will be *much* shorter than this practice exam. This practice exam is mainly here to give you plenty of opportunity to reinforce your calculus skills.

1. **Simple(r) U-subs**. Compute the following integrals:

   (a) $\int_0^{1/2} \cos(3\pi x)\, dx$

   (b) $\int e^{12\pi x}\, dx$

   (c) $\int_5^6 (x-5)^3\, dx$

   (d) $\int \frac{1}{1+(x-10)^2}\, dx$

2. Find a value of c such that $\int_0^1 c\sin(cx)\, dx = 2$ (note: there are multiple answers here – you only need to find one of them).

3. **Fundamental Theorem of Calculus**. Find the derivative of the following functions:

   (a) $F(x) = \int_{-1}^{x} \sqrt{1+\cos(t)}\, dt$

   (b) $F(x) = \int_0^{x^3} e^{t^2}\, dt$

   (c) $F(x) = \int_{\tan(x)}^{50} \ln(|t|)\, dt$

   (d) $F(x) = \int_{-1000}^{2\ln(\pi)} \cos(50t+t^2)e^{\sqrt{t}}(1+\tan(t))\, dt$

4. Suppose the velocity of a car at time t is given by the function $v(t) = t^3 + \cos(t)$. Assuming the position of the car at time $t=0$ is 5, find the position of the car at time $t=2$ (you do not need to simplify any trig functions in your final answer).

5. **Advanced U-subs**. Compute the following integrals:

   (a) $\int x^3\cos(5x^4)\, dx$

   (b) $\int 3\sec^2(x)e^{\tan(x)}\, dx$

   (c) $\int x^2\sqrt{5+x^3}\, dx$

   (d) $\int \cot(x)\, dx$

(e) $\int \cos^{2025}(x)\sin(x)\,dx$

6. **Area Between Curves.** Find the area of the described regions:
   (a) The region bounded by the lines $x = 1$ and $x = 2$ and the curves $y = \sqrt{x}$ and $y = \ln x$ (hint: finding out which function is the top and which is the bottom might be tricky. Try plugging in $x = 1$ and $x = 2$ into both functions, recalling that $\ln 2 \approx 0.693$).
   (b) The region enclosed by the line $y = 2$ and the curves $y = e^{\frac{1}{2}x}$ and $y = e^{-\frac{1}{2}x}$.
   (c) The region enclosed by the line $y = x$ and the parabola $y^2 = 4x + 5$.
   (d) The region bounded by the lines $x = 0$, $x = 4$, and the curves $y = \cos(x)$ and $y = \sin(x + \pi)$ (hint: $\sin(x + \pi) = -\sin(x)$).

7. **Volumes**. Find the volume of the described solids:
   (a) The solid obtained by rotating the region bounded by the curve $y = e^x$ and the lines $y = 1$ and $x = 1$ about the x-axis.
   (b) The solid obtained by rotating the region bounded by the curve $y = 3\sqrt{x}$ and the lines $y = 0$ and $x = 1$ about the line $x = 2$.
   (c) The solid obtained by rotating the region in the first quadrant bounded by the curve $y = e^{x^2}$ and the line $y = e$ about the y-axis.

8. **Integration by Parts**. Compute the following integrals:
   (a) $\int_0^1 x^2 \sin(\pi x)\,dx$
   (b) $\int e^{3x}\sin(2x)\,dx$
   (c) $\int (\ln x)^2\,dx$
   (d) $\int_1^e \frac{\ln(x)}{x^5}\,dx$

9. **Advanced Trig Integrals**. Compute the following integrals:
   (a) $\int \cos^2(x)\sin^2(x)\,dx$
   (b) $\int \sin^3(3x)\cos^2(3x)\,dx$
   (c) $\int \sin(2x)\cos^2(x)\,dx$
   (d) $\int \tan^5(x)\sec^4(x)\,dx$ (there are two ways to approach this problem)

(e) $\int \sin(3x)\cos(4x)\,dx$

10. **Trig Substitution**. Compute the following integrals:

(a) $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

(b) $\int \frac{\sqrt{25x^2-4}}{x}$

(c) $\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$

(d) $\int \frac{1}{x^4\sqrt{9-x^2}}\,dx$

11. **Freestyle**. Compute the following integrals by any means necessary. Try to use the easiest method(s) available!

(a) $\int_{-2025}^{2025} \cos(x)\sin(x)x^{100}\,dx$

(b) $\int_{-5}^{5} 3|x|\,dx$

(c) $\int \frac{x^3}{1+x^2}\,dx$

(d) $\int \frac{x^2}{1+x^2}\,dx$

(e) $\int x^3 e^{x^2}\,dx$ (hint: remember that e^{x^2} is impossible to integrate, but xe^{x^2} is possible)

(f) $\int \frac{5x^2}{1+x^6}\,dx$